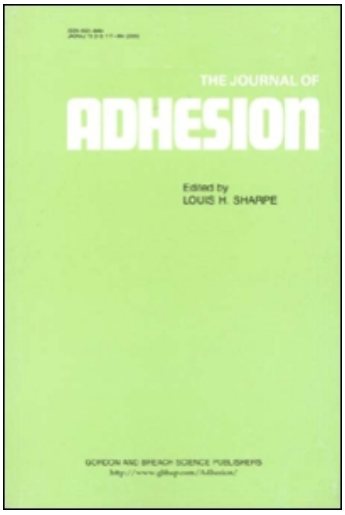

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The Shapes of Peeling Solid Films

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The Shapes of Peeling Solid Films

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A theory is developed to describe the shape of a long thin elastic film peeling from a rigid substrate under a constant load. The theory is verified by static measurements of spring steel strip and by slow peeling experiments using rubber and paint films. Extension of the theory to the case of viscoelastic peeling films demonstrates how the film shape in such a system depends on peeling velocity. Reasonable agreement with experiment is obtained.

I INTRODUCTION

The thin film peeling test (Figure 1) has been used for many years in studies of the adhesion of materials. In particular, much attention has been given to the peel strength of adhesive tape systems where a polymeric film is sandwiched between two solid sheets which are then peeled apart.[1,3,6,11] The

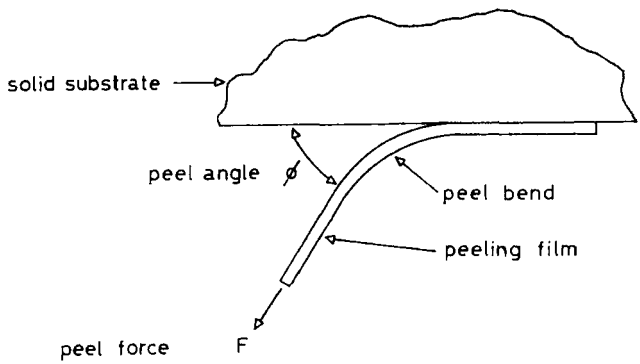

FIGURE 1 The thin film peeling test.

shape of such peeling joints has been explained in terms of the rheological properties of the interlayer.

The simpler configuration of Figure 1, where molecular contact between an elastic solid and a rigid substrate is broken by peeling, has been less comprehensively treated.[2,7,8] Clearly this peeling geometry is merely an extension of the cantilever test used in the fracture experiments of Obreimoff[9] and Gilman.[4] Essentially the same theory can be applied to both the cantilever and peeling arrangements—energy is assumed to be conserved as the surfaces are separated. In effect the method of Griffith[5] is employed.

To compute the energy changes as peeling occurs it is necessary to know the shape of the peeling film. This paper considers first the shape of elastic films peeling from rigid substrates. The theory is verified by measurements of steel and polymeric strips. The argument is then extended to the peeling shapes of viscoelastic materials.

II THEORY FOR ELASTIC FILMS

The geometry considered is shown in Figure 2. An elastic film is pulled from a rigid surface AB by a peel force F acting along the x axis. The y axis is chosen to pass through the point of separation of the surfaces at $y = l$. The peeling film makes an angle of θ_l with the x axis at this point, the peel angle ϕ being given by the equation

$$\phi = \frac{\pi}{2} - \theta_l \tag{1}$$

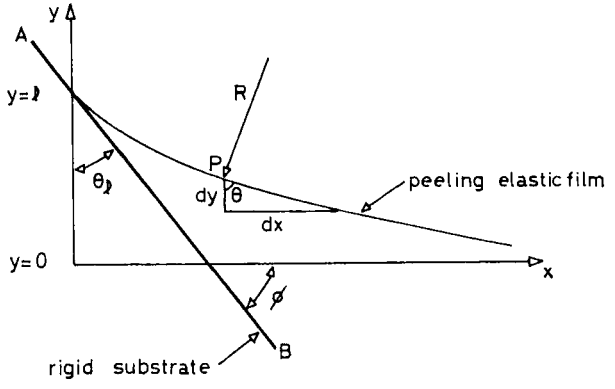

FIGURE 2 Geometry of a peeling film.

At a large distance from the y axis the peeling film is coincident with the x axis. Gravitational forces acting on the film are assumed to be negligible.

At a point P on the peeling film where the radius of curvature is R, the bending equation may be applied

$$\frac{EI_A}{R} = Fy \tag{2}$$

where E is the Young's modulus of the elastic material and I_A is the second moment of area about the neutral axis of the film. Providing that the elastic film is thin, displacements of the film due to shear may be neglected.

$$\frac{\dfrac{d^2x}{dy^2}}{\left[1 + \left(\dfrac{dx}{dy}\right)^2\right]^{3/2}} = \frac{Fy}{EI_A} \tag{3}$$

$$p = \frac{dx}{dy} = -\tan\theta \tag{4}$$

$$\frac{dp}{dy}\frac{1}{(1+p^2)^{3/2}} = \frac{F}{EI_A}y \tag{5}$$

$$-\sin\theta = \frac{F}{EI_A}\frac{y^2}{2} + C \tag{6}$$

where C is an arbitrary constant which may be evaluated from the condition that $y = l$ when $\theta = \theta_l$

$$\sin\theta - \sin\theta_l = \frac{F}{2EI_A}(l^2 - y^2) \tag{7}$$

Also, since at $y = 0$, $\sin\theta = 1$

$$l^2 = \frac{2EI_A}{F}(1 - \sin\theta_l) \tag{8}$$

From Eq. (6)

$$\sin\theta = \frac{F}{2EI_A}(l^2 - y^2) + \sin\theta_l \tag{9}$$

$$= \frac{F}{2EI_A}(l^2 - y^2 - l^2) + 1 \tag{10}$$

$$= 1 - \frac{y^2}{l^2}(1 - \sin\theta_l) \tag{11}$$

Put

$$r = \frac{y}{l}\sqrt{(1 - \sin\theta_l)} \tag{12}$$

$$\sin\theta = 1 - r^2 \tag{13}$$

$$\tan\theta = -\frac{dx}{dy} = \frac{1 - r^2}{\sqrt{1 - (1 - r^2)^2}} \tag{14}$$

Eq. (14) is readily integrated to give the shape of the peeling film. The final equation is

$$\frac{\sqrt{1 - \sin\theta_l}\, x}{l} = \frac{1}{\sqrt{2}} \ln\left[\frac{\sqrt{2} + \sqrt{2 - \frac{y^2}{l^2}(1 - \sin\theta_l)}}{\frac{y}{l}(\sqrt{2} + \sqrt{1 + \sin\theta_l})}\right] + \sqrt{1 + \sin\theta_l} - \sqrt{2 - \frac{y^2}{l^2}(1 - \sin\theta_l)} \tag{15}$$

It is interesting to note that this equation also describes the shape of two-dimensional liquid menisci such as those at large drops, bubbles and Wilhelmy plates.[10]

III EXPERIMENT

To test Eq. (15) a spring steel strip 0.60 m long, 0.019 m wide and 0.258 mm thick was used. One end of the strip was cemented with Araldite into a slot machined in a massive steel block. At the other end of the strip, a spring balance was attached and a force applied to bend the strip in a horizontal plane. The angle ϕ was set at a suitable value and the force adjusted until l became equal to 0.075 m. The measured shapes are given in Figure 3 for four values of the peel angle ϕ. Reasonable agreement with Eq. (15) was obtained. Slight deviations from the theoretical shape were noticed, particularly where ϕ was equal to π. It appeared that the effective peel angle was slightly less than the desired angle. The discrepancy amounted to about 2° and was perhaps due partly to some deformation of the joint between the steel strip and the massive steel block. When this joint was formed by bolting rather than glueing the discrepancy increased to about 4°. The value of Young's modulus calculated from Eq. (8) was found to be $2.14 \pm .05 \times 10^{11}$ Nm^{-2} rather lower than the figure of $2.20 \pm .01 \times 10^{11}$ Nm^{-2} found by cantilever bending measurements. Once again this low value indicates that the effective peel angle is slightly less than the nominal angle.

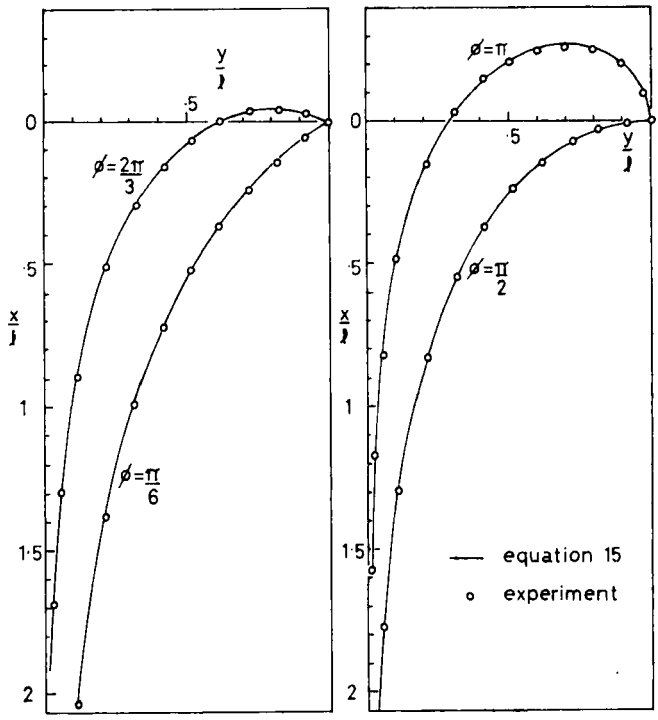

FIGURE 3 Results for the shape of a bent steel strip.

The shapes of peeling polymeric films were also studied, the peel angle being maintained at $\pi/2$. Silicone rubber films peeling from poly(methylmethacrylate) (I.C.I. Perspex) and uroalkyd paint films peeling from glass were studied. New Perspex samples were used, the surfaces being washed gently in warm water and detergent and rinsed for some time in tap water. The glass surfaces were vapour degreased in isopropanol.

A film of the liquid polymer was spread to a uniform thickness on a sheet of optically smooth glass or Perspex and allowed to cure at room temperature. After curing and stripping, the long-time elastic modulus of each polymer was measured in a bending experiment and was found to be $(8.3 \pm .5) \times 10^6\ \mathrm{Nm}^{-2}$ for the silastomer and $(1.08 \pm .1) \times 10^7\ \mathrm{Nm}^{-2}$ for the paint at 18°C.

A load was attached to a partially peeled, one cm wide strip of the polymer and peeling was allowed to proceed under gravity at a slow rate (peel velocity less than 1 $\mu\mathrm{m\ sec}^{-1}$). To facilitate measurement of the bend shape the parameter l was arranged to be several mm, the load required to produce this

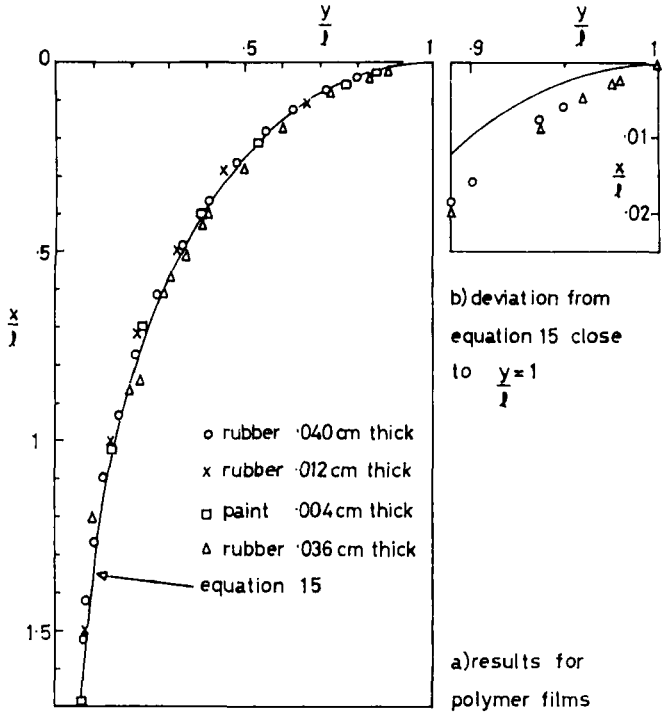

FIGURE 4 Results for the shape of slowly peeling polymer films.

being between 0.1 and 1 gm. The peeling films were photographed and measured and the results given in Figure 4a show that agreement with Eq. (15) is reasonable at some distance from the point where separation of the polymer film from the substrate just occurs. Near this point a significant difference between experiment and theory was observed. This discrepancy is shown in Figure 4b where microscope measurements of a rubber film are compared with the solid line given by Eq. (15). The experimental points deviated considerably from the simple theory, as though the peel angle was smaller than its nominal value by about 7°. When the rubber film was clamped between two glass plates and the bending remeasured, a similar deviation from the theory was found.

IV VISCOELASTIC FILMS

The theory given above corresponds only to perfectly elastic films where Young's modulus is constant. If the elastic modulus varies with time as in

the case of viscoelastic polymer films, the peel shape will then depend on the time available for relaxation of the modulus, that is, on the speed of peeling. In this section the effect of an idealised relaxation (Figure 5) on the shape of a steadily peeling film will be studied. At short times the Young's modulus of the peeling film is assumed to be E_0. After a time τ the modulus is assumed to fall sharply to $0.2E_0$ and remains steady at this value for longer times. The model used here is quite arbitrary.

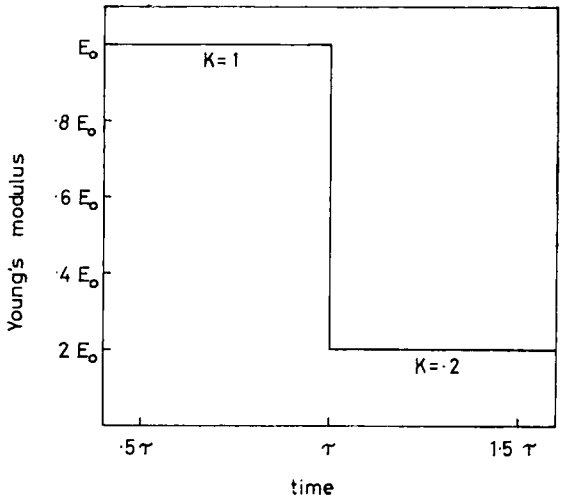

FIGURE 5 Idealised relaxation in a viscoelastic film.

A numerical method was adopted for calculating the shapes of the peeling films. Figure 6 shows the essential features of the method. The viscoelastic film, assumed to be peeling at a steady velocity V, was divided into an arbitrary number of equal elements ds long, corresponding to equal time increments. In this case 200 elements were found sufficient to give adequate accuracy. The $n + 1^{th}$ coordinates of the film were then calculated from the n^{th} coordinates using the equations

$$y_{n+1} = y_n - ds \cos \theta_n \tag{16}$$

$$x_{n+1} = x_n + ds \sin \theta_n \tag{17}$$

$$\theta_{n+1} = \theta_n + \frac{ds}{R_n} \tag{18}$$

The radius of curvature of the bending film at each point depends on the strain history of the element and may be calculated from the Boltzmann superposition principle (see, for example, Tobolsky[12]) expressed in the

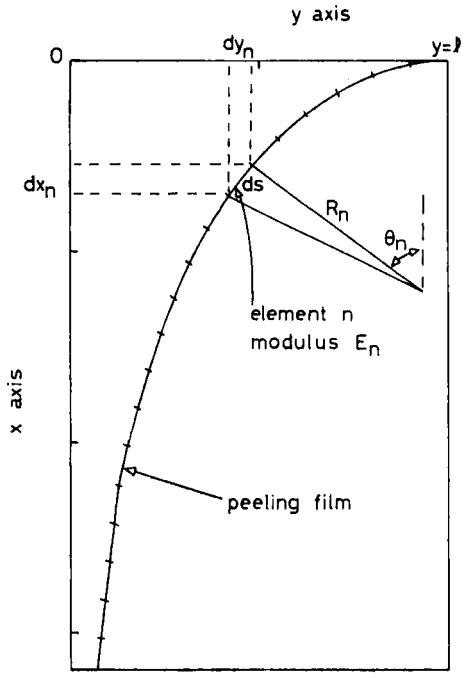

FIGURE 6 Numerical determination of the peel bend shape.

following form.

$$\frac{1}{R_n} = \frac{F}{I_A}\left[\frac{l}{E_n} - \sum\left(\frac{dy_1}{E_n} + \frac{dy_2}{E_{n-1}} + \cdots + \frac{dy_n}{E_1}\right)\right] \tag{19}$$

Eqs. (16) to (19) may be expressed in terms of the non-dimensional variables a, b, c and K_n defined below.

$$a = \frac{y}{l} \tag{20}$$

$$b = \frac{x}{l} \tag{21}$$

$$K_n = \frac{E_n}{E_0} \tag{22}$$

$$c = \frac{s}{l} \tag{23}$$

Combined with Eqs. (16) to (19) we have

$$a_{n+1} = a_n - dc \cos \theta_n \tag{24}$$

$$b_{n+1} = b_n + dc \sin \theta_n \tag{25}$$

$$\theta_{n+1} = \theta_n - dc \frac{l}{R_n} \tag{26}$$

$$\frac{l}{R_n} = \frac{Fl^2}{I_A E_0}\left[\frac{1}{K_n} - \sum\left(\frac{da_1}{K_n} + \frac{da_2}{K_{n-1}} + \cdots + \frac{da_n}{K_1}\right)\right] \tag{27}$$

Using the values of K shown in Figure 5 the shapes of the films were computed for a range of peeling velocities, the value of the parameter $Fl^2/I_A E_0$ being adjusted in each case by trial and error until the film approached the b axis at $a = 0$.

The film shapes for five different steady peeling velocities are shown in Figures 7 and 8. At low speeds (Figure 7a) relaxation occurs before the film has travelled far into the peel bend and, consequently, the shape is not significantly different from that of Figure 4. Again at very high speeds (Figure

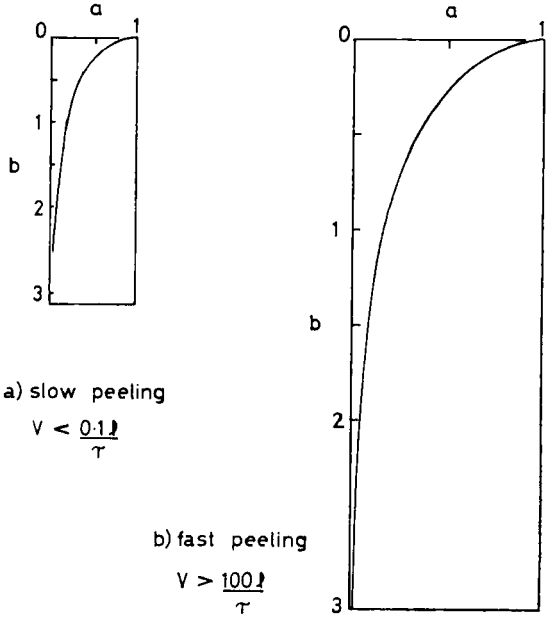

FIGURE 7 Predicted shapes of idealised viscoelastic films peeling at low and high speeds under the same force.

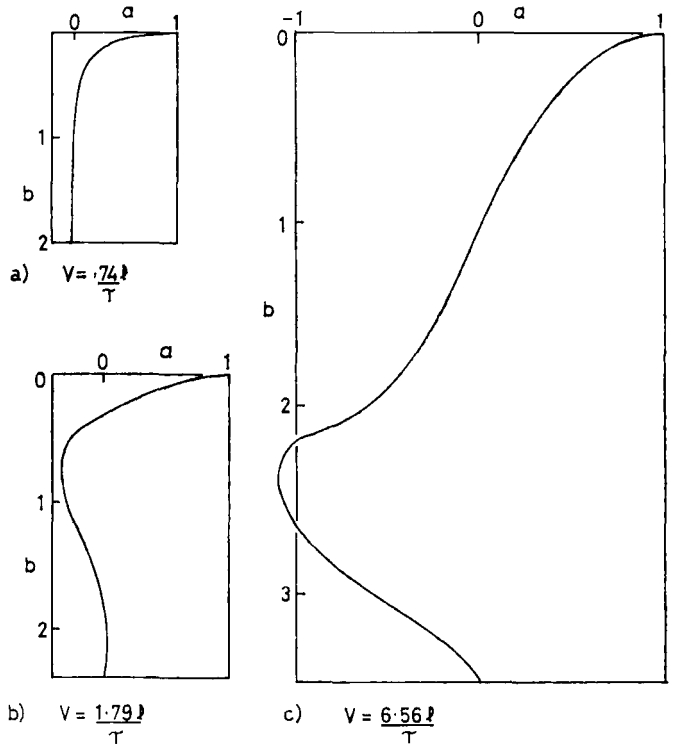

FIGURE 8 Predicted shapes of idealised viscoelastic films at intermediate speeds.

7b) the relaxation takes place after the film has traversed the peel bend and the shape is the same but with a higher value of l corresponding to the larger short time modulus of the film material. Figure 7 has been plotted to show this increase in l at high speeds assuming that the peel force remains constant throughout the speed range. In practise an increase in peel force is usually required at higher speeds and this tends to reduce the value of l. When peeling takes place at intermediate velocities, relaxation arises in the middle of the bend and some interesting shapes are produced. Figure 8 demonstrates that at certain speeds the film may overshoot the b axis, a fact which has been experimentally observed in the past, though no published reference is known to me.

V EXPERIMENT

Peeling experiments were carried out to discover whether the curious shapes predicted by the idealised analysis could arise in practise. A uroalkyd paint

film 0.004 cm thick and 1 cm wide was pulled from a glass surface which had been precoated with dimethyldichlorosilane to reduce the adhesive force. The peeling film was photographed and measured. A typical result, where the peel

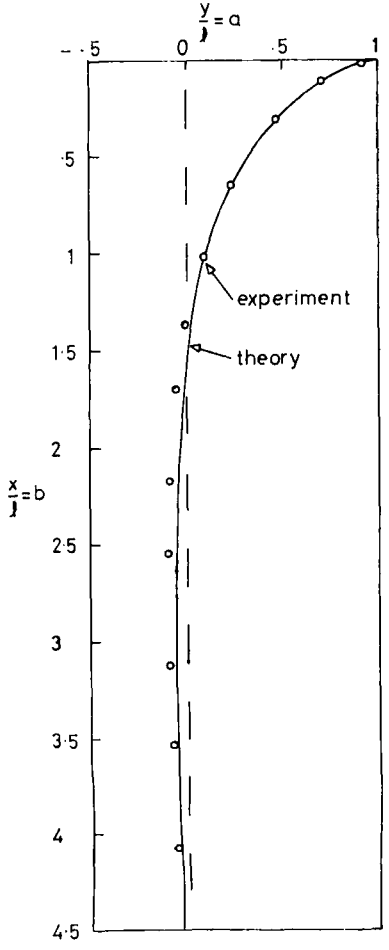

FIGURE 9 The shape of a peeling paint film.

velocity was 0.083 cm sec^{-1} and the peel angle 7.1° is shown by the points in Figure 9. Qualitatively, this experiment did confirm the idealised theory. The film crossed the b axis and then gradually returned. The effect was not very marked, presumably because the elastic modulus of the paint film did not relax suddenly as assumed in Figure 8. The actual fall in modulus of the paint film with time, when measured in a tensile stress relaxation test, showed

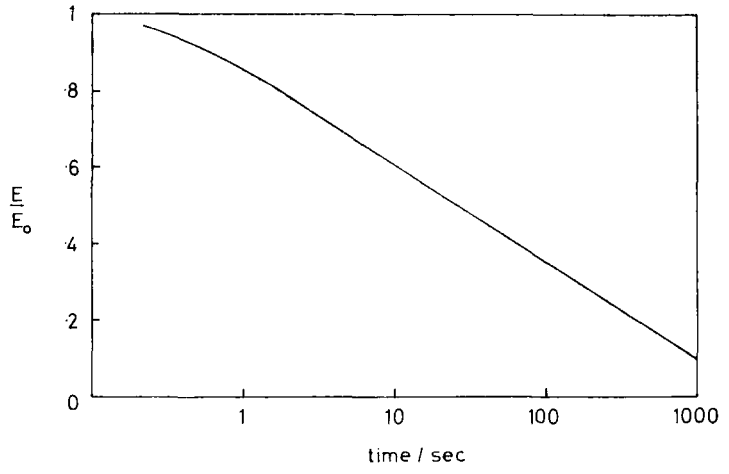

FIGURE 10 Relaxation of paint film elastic modulus with time at 20°C.

the behaviour of Figure 10. Using these experimental values of elastic modulus in Eqs. (20) to (27) the theoretical shape was computed and is shown for comparison with the experimental shape in Figure 9. Agreement between the experimental and theoretical results was encouraging.

VI CONCLUSION

A theory has been developed to describe the shapes of peeling solid films. Films of elastic material, and also viscoelastic films peeling at very high or very low velocities, are described by a simple bending theory. At intermediate peeling rates viscoelastic films relax as they traverse the peel bend and these relaxations of modulus may be incorporated into the bending theory to enable the film shape to be determined numerically. Experimental measurements of peeling films of rubber and paint give reasonable agreement with the theory. This work should be useful to the study of thin film adhesion.

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